Use an appropriate linear approximation to estimate $\tan^{-1} 0.97$.

SCORE: ____ / 5 PTS

$f(x) = \tan^{-1} x$, $a = 1$

$f(x) \approx f(a) + f'(a)(x-a)$ FOR $x \approx a$

$= \tan^{-1} 1 + \frac{1}{1+1^2}(x-1)$ FOR $x \approx 1$

$= \frac{\pi}{4} + \frac{1}{2}(x-1)$

$\tan^{-1} 0.97 \approx \frac{\pi}{4} + \frac{1}{2}(0.97-1) = \boxed{\frac{\pi}{4}} \; \boxed{+ \frac{1}{2}} \; \boxed{(-0.03)} = \boxed{\frac{\pi}{4} - 0.015}$

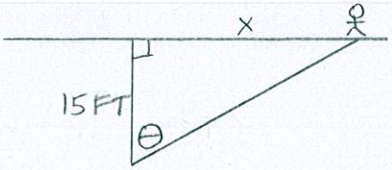
(1) (1½) (1½) (1)

A man walks along a straight path. A searchlight is located on the ground 15 feet from the path, and is kept focused on the man. When the man is 20 feet from the point on the path closest to the searchlight, the light is turning at 3° per second. How quickly is the man walking at that moment ?

SCORE: ____ / 10 PTS

You must state/show clearly what each variable you use represents.

You must show the units during the intermediate steps of your work, and you must state the units for the final answer.



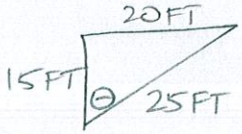
$\left.\frac{d\theta}{dt}\right|_{x=20\,FT} = 3^\circ/SEC = \frac{\pi}{60}$ RADIANS/SEC OR $\frac{\pi}{60}$/SEC

WANT $\left.\frac{dx}{dt}\right|_{x=20\,FT}$

(2) $\frac{X}{15\,FT} = \tan\theta$

(1) $\frac{1}{15\,FT}\frac{dx}{dt} = \sec^2\theta\frac{d\theta}{dt}$ (1)

(a) $X = 20\,FT$ $\quad \frac{1}{15\,FT}\frac{dx}{dt} = \left(\frac{25\,FT}{15\,FT}\right)^2 \frac{\pi}{60}$/SEC



$\frac{dx}{dt} = (15\,FT)\left(\frac{5}{3}\right)^2 \frac{\pi}{60}$/SEC (2) (1½)

$= \frac{25\pi}{36}$ FT/SEC (1)

(½) THE MAN IS WALKING AT $\frac{25\pi}{36}$ FEET PER SECOND

Find the values of Δy and dy if $y = \frac{1}{\sqrt{x}}$ and x changes from 1 to 4.

SCORE: _____ / 5 PTS

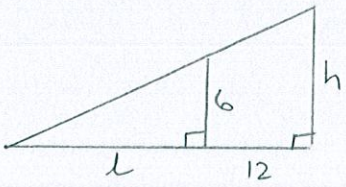
$$\Delta y = y(4) - y(1) = \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{1}} = \boxed{\frac{1}{2} - 1} = \boxed{-\frac{1}{2}}$$

(1) (½)

$$dy = y'(1)\,dx = \boxed{-\frac{1}{2}(1)^{-\frac{3}{2}}}\boxed{(4-1)} = \boxed{-\frac{3}{2}}$$

(1½) (1½) (½)

A 6 foot tall woman is standing 12 feet from a pole. A spot light is being lowered from the top of the pole. **SCORE: _____ / 10 PTS**
At the moment when the light is 15 feet from the ground, the light is moving 2 feet per second down the pole.
How quickly is the length of the woman's shadow changing at that moment ?
You must state/show clearly what each variable you use represents.
You do NOT need to show the units during the intermediate steps of your work, but you must state the units for the final answer, and whether the shadow is getting longer or shorter.



$\left.\frac{dh}{dt}\right|_{h=15\,FT} = -2\ FT/SEC$

WANT $\left.\frac{dl}{dt}\right|_{h=15\,FT}$

$\frac{l}{6} = \frac{l+12}{h}$ ⟶ @ $h=15$ FT $\quad \frac{l}{6} = \frac{l+12}{15}$

(2) $lh = 6l+72$ ← TALK TO ME IF YOU WROTE THIS EQUATION IN ANOTHER WAY

$15l = 6l+72$
$9l = 72$
$l = 8$

(1) $\frac{dl}{dt}h + l\frac{dh}{dt} = 6\frac{dl}{dt}$ (1/2)

$\frac{dl}{dt}(15) + 8(-2) = 6\frac{dl}{dt}$
(1) (1/2) (1/2)

$9\frac{dl}{dt} = 16$

(1) $\frac{dl}{dt} = \frac{16}{9}$

(1/2)
THE SHADOW IS GETTING LONGER
BY $\frac{16}{9}$ FEET PER SECOND
(1/2) (1/2)